NAME ______________________ DATE ____________

9–2 The Substitution Method

Objective: To use the substitution method to solve systems of linear equations.

Example 1 Solve by the substitution method: $x + y = 9$, $2x + 3y = 20$

Solution

1. Solve the first equation for y.

$$x + y = 9$$
$$y = 9 - x$$

2. Substitute this expression for y in the other equation, and solve for x.

$$2x + 3(9 - x) = 20$$
$$2x + 27 - 3x = 20$$
$$-x + 27 = 20$$
$$-x = -7$$
$$x = 7$$

3. Substitute the value for x in the equation in Step 1, and solve for y.

$$y = 9 - x$$
$$y = 9 - 7$$
$$y = 2$$

4. Check $x = 7$ and $y = 2$ in *both* equations.

$$x + y = 9 \qquad 2x + 3y = 20$$
$$7 + 2 \stackrel{?}{=} 9 \qquad 2(7) + 3(2) \stackrel{?}{=} 20$$
$$9 = 9 \checkmark \qquad 14 + 6 \stackrel{?}{=} 20$$
$$20 = 20 \checkmark$$

The solution is (7, 2).

Solve by the substitution method.

1. $y = 3x$; $x + y = 12$ **(3, 9)**
2. $y = 2x$; $5x - y = 12$ **(4, 8)**
3. $a = 4b$; $a - b = 9$ **(12, 3)**
4. $m = 5n$; $3m - 2n = 26$ **(10, 2)**
5. $y = x - 1$; $2x + y = 5$ **(2, 1)**
6. $y = 4x - 1$; $x + y = 4$ **(1, 3)**
7. $x + y = 3$; $2x - y = 6$ **(3, 0)**
8. $x - y = 2$; $x - 2y = -1$ **(5, 3)**
9. $3x - y = -9$; $4x + y = -5$ **(−2, 3)**
10. $2x + y = 1$; $3x + 2y = 3$ **(−1, 3)**
11. $3x + y = 7$; $2x - 5y = -1$ **(2, 1)**
12. $x - 3y = -5$; $2x - 5y = -9$ **(−2, 1)**
13. $4x - 2y = 5$; $x - 4y = 3$ $\left(\mathbf{1, -\frac{1}{2}}\right)$
14. $2x + y = 3$; $3x + 2y = 5$ **(1, 1)**
15. $3y - x = -8$; $5y + 2x = -6$ **(2, −2)**
16. $3x + y = 2$; $2x + 3y = -8$ **(2, −4)**
17. $x + 2y = 7$; $2x - y = 4$ **(3, 2)**
18. $x - 3y = 2$; $x = -y - 6$ **(−4, −2)**
19. $x - 5 = y$; $5x + 2y = 4$ **(2, −3)**
20. $y - 3 = -2x$; $3x - 2y = -20$ **(−2, 7)**
21. $x + 8 = 2y$; $4x + y = 13$ **(2, 5)**
22. $3u + v = 8$; $\frac{u}{4} - \frac{v}{2} = 3$ **(4, −4)**
23. $2x - y = 2$; $x = \frac{2}{3}y$ **(4, 6)**
24. $5x - 4y = -10$; $x = \frac{3}{5}y$ **(6, 10)**

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9–2 The Substitution Method *(continued)*

Example 2 Solve by the substitution method: $2x - 6y = 8$, $x - 3y = 10$

Solution

$$x - 3y = 10$$
$$x = 10 + 3y$$

$$2x - 6y = 8$$
$$2(10 + 3y) - 6y = 8$$
$$20 + 6y - 6y = 8$$
$$20 = 8 \leftarrow \text{False}$$

The *false statement* indicates that there is *no* ordered pair (x, y) that satisfies both equations. (If you graph the equations, you'll see that *the lines are parallel.*)

The system has *no solution.*

Example 3 Solve by the substitution method: $\frac{y}{3} = 3 - x$, $3x + y = 9$

Solution

$\frac{y}{3} = 3 - x$ Multiply both sides by 3 to solve for y.

$$y = 9 - 3x$$

$$3x + y = 9$$
$$3x + (9 - 3x) = 9$$
$$3x + 9 - 3x = 9$$
$$9 = 9 \leftarrow \text{True}$$

The *true statement* indicates that *every* ordered pair (x, y) that satisfies one of the equations also satisfies the other. (If you graph the equations, you'll see that *the lines coincide.*)

The system has *infinitely many solutions.*

Solve by the substitution method.

25. $x - 3y = -2$; $y = 2x - 1$ **(1, 1)**
26. $x + 2y = 7$; $2x + 4y = 8$ **No solution**
27. $y = 2x - 3$; $2y = -3x + 8$ **(2, 1)**
28. $\frac{x}{2} = 3 - y$; $x + 2y = 6$ **Infinitely many solutions**
29. $9x - 5y = 105$; $\frac{1}{4}x - \frac{2}{5}y = -1$ **(20, 15)**
30. $\frac{x}{3} = 2 + y$; $3x - 9y = -4$ **No solution**

Mixed Review Exercises

Write an equation in slope-intercept form for each line described.

1. slope $\frac{1}{2}$, passes through $(-2, 4)$ $\mathbf{y = \frac{1}{2}x + 5}$
2. slope $\frac{2}{3}$, passes through $(3, -3)$ $\mathbf{y = \frac{2}{3}x - 5}$
3. slope 3, y-intercept 2 $\mathbf{y = 3x + 2}$
4. passes through (2, 7) and (0, −3) $\mathbf{y = 5x - 3}$
5. passes through (2, −4) and (−1, 1) $\mathbf{y = -\frac{5}{3}x - \frac{2}{3}}$
6. slope 0, y-intercept −3 $\mathbf{y = -3}$